Linear Constant co-efficient difference eq'n

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Causal system

$$\sum_{k=0}^{\infty} x(k)\, h(n-k)$$

$$\sum_{k=0}^{\infty} h(k)\, x(n-k)$$

Depending upon length of impulse response two types

1. Finite impulse response (FIR)
2. Infinite impulse response (IIR)

FIR   Impulse fn have finite length assume $-M$ to M

$$y(n) = \sum_{k=-M}^{M} h(k)\, x(n-k)$$ non causal

$$y(n) = \sum_{k=0}^{M} h(k)\, x(n-k)$$ Causal

IIR $h(n)$ is infinite

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$ Non Causal

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$ causal

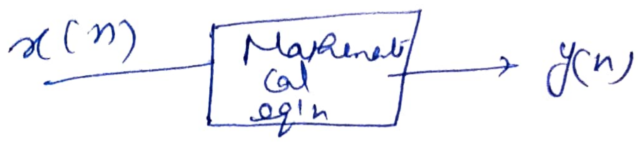
depending upon length of $h(n)$ we have classified the system FIR & IIR. Similarly depending upon output of system. DT system is classified as

1. Non - recursive system
2. Recursive systems.

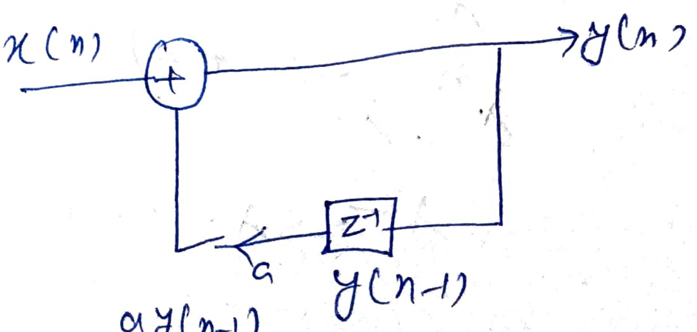
Non- Recursive → not required any past O/P sample to calculate the present output

$$y(n) = \sum_{k=0}^{M} h(k)\, x(n-k)$$ Causal recursive

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$ Non causal recursive not possible



Recursive System → O/P depends on present I/P & past I/P as well as previous O/P



$y(1) = x(1) + a\,y(0)$
Pres Pres - Past

$y(n) = x(n) + a\,y(n-1)$

Past $y(0) = x(0) + a\,y(-1)$
Pres Present Past

Determine the homogeneous solution of the system

$$y(n) - 3y(n-1) - 4y(n-2) = x(n)$$

Step 1 Assume input $x(n) = 0$

$$y(n) - 3y(n-1) + 4y(n-2) = \cancel{x(n)}\ 0 \quad \text{—— (1)}$$

Step 2 Assume the homogeneous soln

$$y_h(n) = \lambda^n \quad \text{—— (2)}$$

$$y(n-1) = \lambda^{n-1} \qquad y(n-2)\ \lambda^{n-2}$$

Step 3

$$\lambda^n - 3\lambda^{n-1} + 4\lambda^{n-2} = 0 \quad \text{—— (3)}$$

Taking λ^{n-2} common

$$\lambda^{n-2}\left[\frac{\lambda^n}{\lambda^{(n-2)}} - \frac{3\lambda^{n-1}}{\lambda^{(n-2)}}\right]$$

$$\lambda^{n-2}\left[\lambda^n \cdot \lambda^{-n+2} - 3\lambda^{n-1} \cdot \lambda^{-n+2} + 4\right] = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 3\lambda - 4\right] = 0$$

$$\left[\lambda^2 - 3\lambda - 4\right] = 0 \quad \text{—— (4)}$$

Now obtain roots of eq'n

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{3 \pm \sqrt{9 - 4(1)(-4)}}{2 \times 1} = \frac{3 \pm \sqrt{25}}{2}$$

$$\lambda_1 = \frac{3-5}{2} = -1 \qquad \lambda_2 = \frac{3+5}{2} = 4$$

<u>step 4</u> homogeneous soln is given by

$$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n$$

$$y_h(n) = C_1(-1)^n + C_2(4)^n \quad —(5)$$

<u>step 5</u> from eq (5)

$n=0 \quad y(0) = C_1 + C_2 \quad —(6)$

$n=1 \quad y(1) = -C_1 + 4C_2 \quad —(7)$

<u>step 6</u> Now taking original eq'n

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

$$y(n) = 3y(n-1) + 4y(n-2) \quad —(8)$$

$n=0 \Rightarrow y(0) = 3y(-1) + 4y(-2) \quad —(9)$

$n=1 \quad y(1) = 3y(0) + 4y(-1) \quad —(10)$

Put eq (9) in eq (10)

$$y(1) = 3[3y(-1) + 4y(-2)] + 4y(-1)$$

$$9y(-1) + 12y(-2) + 4y(-1)$$

$$y(1) = 13y(-1) + 12y(-2) \quad —(11)$$

<u>step 7</u> Equate eq^n $n=0$ mean eq (6) & (9)

$$C_1 + C_2 = 3y(-1) + 4y(-2) \quad —(12)$$

And eq (7) & (11)

$$-C_1 + 4C_2 = 13y(-1) + 12y(-2) \quad —(13)$$

Add eq (12) & (13)

$$5C_2 = 16y(-1) + 16y(-2)$$

$C_2 = \frac{16}{5}y(-1) + \frac{16}{5}y(-2)$ — 14

Put this value in eq^n 12

$C_1 + \frac{16}{5}y(-1) + \frac{16}{5}y(-2) = 3y(-1) + 4y(-2)$

$C_1 = 3y(-1) - \frac{16}{5}y(-1) + 4y(-2) - \frac{16}{5}y(-2)$

$C_1 = -\frac{1}{5}y(-1) + \frac{4}{5}y(-2)$

Step5 Put these values in eq^n 5

$y_h(n) = C_1(-1)^n + C_2(4)^n$

$\left[-\frac{1}{5}y(-1) + \frac{4}{5}y(-2)\right](-1)^n + \left[\frac{16}{5}y(-1) + \frac{16}{5}y(-2)\right](4)^n$

Assume $y(-1)$ & $y(-2)$ initials conditions are zero

$y_h(n) = (-1)^n + (4)^n$

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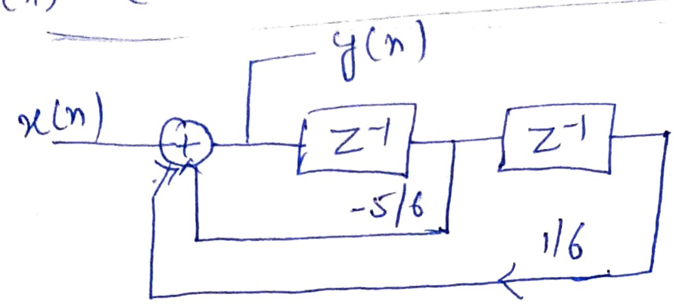

$y(n) = x(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$ — 1

Step1 $y(n) + \frac{5}{6}y(n-1) \mp \frac{1}{6}y(n-2) = 0$ — 2

Step2 $y_h(n) = \lambda^n$

Step3 $\lambda^n + \frac{5}{6}\lambda^{n-1} \mp \frac{1}{6}\lambda^{n-2} = 0$ — 3

Taking λ^{n-2} common

$$\lambda^{n-2}\left[\lambda^{n}\cdot\lambda^{-n+2}+\frac{5}{6}\lambda^{n-1}\cdot\lambda^{-n+2}-\frac{1}{6}\right]=0$$

$$\lambda^{n-2}\left[\lambda^{2}+\frac{5}{6}\lambda-\frac{1}{6}\right]=0$$

$$\therefore \lambda^{2}+\frac{5}{6}\lambda-\frac{1}{6}=0 \quad — (4)$$

roots

$$\lambda=\frac{-\frac{5}{6}\pm\sqrt{\frac{25}{36}-\left[4\times1\times\left(-\frac{1}{6}\right)\right]}}{2}$$

$$\lambda=\frac{-\frac{5}{6}\pm\sqrt{\frac{25}{36}+\frac{4}{6}}}{2}=\frac{-\frac{5}{6}\pm\sqrt{\frac{25+24}{36}}}{2}=\frac{-\frac{5}{6}\pm\frac{7}{6}}{2}$$

$$\lambda_1=\frac{1}{6}\text{ and }\lambda_2=1$$

Step 4 $\quad y_h(n)=C_1\lambda_1^{n}+C_2\lambda_2^{n}$

$$y_h(n)=C_1\left(\frac{1}{6}\right)^{n}+C_2(-1)^{n} \quad — (5)$$

We will denote $y_h(n)$ by $h(n)$

Step 5 $\quad$ will apply initial conditions

$x(n)=\delta(n) \qquad y(n)=h(n)$

Putting values eq^n (1)

$$h(n)=\delta(n)-\frac{5}{6}h(n-1)+\frac{1}{6}h(n-2)$$

For $n=0 \Rightarrow h(0)=\delta(0)-\frac{5}{6}h(-1)+\frac{1}{6}h(-2)$ $\qquad \delta(0)=1$, $h(-1)=0$

$\delta(0)=1$ initial conditions are zero thus $h(-1)=0$ $h(-2)=0$

$$h(0)=1-\frac{5}{6}\times0+\frac{1}{6}\times0 \Rightarrow \boxed{h(0)=1}$$

~~Step 6~~ $n = 1 \Rightarrow \quad h(1) = \delta(1) - \frac{5}{6} h(0) + \frac{1}{6} h(-1)$

$$h(1) = 0 - \frac{5}{6} \times 1 + \frac{1}{6} \times 0$$

$$h(1) = \frac{-5}{6}$$

Step 6 Put $n = 0$ & (1) in eq (5)

$$h(n) = C_1 \left(\frac{1}{6}\right)^0 + C_2 (-1)^0 = h(0) \quad \text{--- (6)}$$

$$C_1 + C_2 = 1$$

For $n = 1 \Rightarrow \quad C_1 \left(\frac{1}{6}\right)^1 + C_2 (-1)^1 = h(1)$

$$C_1 \cdot \frac{1}{6} - C_2 = \frac{-5}{6} \quad \text{--- (7)}$$

we will obtain C_1 & C_2 from eq (6)

$$C_2 = 1 - C_1$$

Put in eq^n (7)

$$C_1 \cdot \frac{1}{6} - 1 + C_1 = \frac{-5}{6}$$

$$+\frac{7}{6} C_1 = \frac{-5}{6} + 1 = \frac{1}{6}$$

$$7 C_1 = 1 \qquad \boxed{C_1 = \frac{1}{7}}$$

$$C_2 = 1 - C_1 \Rightarrow 1 - \frac{1}{7} = \frac{6}{7} \qquad \boxed{C_2 = \frac{6}{7}}$$

$$h(n) = \frac{1}{7} \left(\frac{1}{6}\right)^n + \frac{6}{7} (-1)^n . \quad \text{Ans}$$